

Pressure of the Standard Model near the electroweak phase transition

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ABSTRACT: We extend our previous determination of the thermodynamic pressure of the Standard Model so that the result can be applied down to temperatures corresponding to the electroweak crossover. This requires a further resummation which can be cleanly organised within the effective theory framework. The result allows for a precise determination of the expansion rate of the Universe for temperatures around the electroweak crossover.

KEYWORDS: Thermal Field Theory, Standard Model.

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1. Introduction

In a recent paper [1] we calculated the pressure of the standard model at high temperatures to three loops, or to order g^5 in the coupling constants. That work followed a long series of calculations dedicated to understanding the perturbative expansion of the pressure of gauge field theories at high temperatures. Especially, within QCD such calculations have been important: the computation of the coefficients of the expansion in g at high temperature has a long history and the result is known today up to the last perturbatively calculable term of order $g^6 \ln g$ [2–4], marking an endpoint to an impressive set of computations carried out in [5–10]. An optimal approach to calculating those coefficients is the effective theory method [11], based on asymptotic freedom and on separating the relevant mass scales: πT , the electric scale $m_E = gT$ and the magnetic scale $m_M = g^2 T$.

An obvious drawback of those calculations within QCD is that they cannot be extended to study the QCD phase transition since the coupling grows large and thus any perturbative calculations become unreliable. That is not the case for the electroweak sector of the standard model. The Landau pole related to the weak interactions corresponds to a length scale $1/\Lambda_{EW} \approx 10^6$ m and therefore the confining effects can be expected to be negligible. As a result, calculating the properties of the electroweak phase transition using, for example, perturbative 1-loop [12–14] and 2-loop [15–17] effective potential calculations was possible for small Higgs masses. A better approach, well defined for large Higgs masses as well, is to perturbatively match the full 4-dimensional theory to an effective 3-dimensional theory [18] and then to numerically solve the phase diagram from the effective theory using

lattice Monte Carlo techniques [19–23]. Such studies show that the phase diagram has a first order line which ends in a 2nd order critical point of Ising universality class [24]. For experimentally allowed Higgs masses the transition is a crossover. Similar techniques have been used to solve the phase diagram also when the external U(1) magnetic field [25] or the chemical potentials related to the baryon and lepton numbers [26] are nonzero. Solving the phase diagram with numerical studies of the full 4-dimensional theory has also been achieved [27].

Although the properties of the electroweak phase transition are known today, computation of the pressure of the theory has been missing. However, the ongoing and future measurements of cosmic microwave background radiation (WMAP and Planck) allow for a precise determination of, for example, the WIMP relic density, which depends on the equation of state of matter in the early universe. A precise determination of the pressure is needed to match the accuracy of the observations [28]. In [1] we performed this calculation when the temperature of the system is high. In the present paper we will extend that result to temperatures close to the critical temperature of the electroweak phase transition. Critical temperature and the phase transition should be understood in perturbation theory framework, where there is always a first order phase transition. We will use these terms throughout this paper, although the actual transition is just a crossover.

The present computation requires a reorganization of the effective theories. Close to the transition the fundamental scalar (which drives the transition) becomes light with respect to the electric scale and thus in a consistent calculation we have to formulate an additional effective theory by integrating out the adjoint scalars. The remaining theory contains just the fundamental scalar and the gauge fields. The pressure will then be composed of three parts: the contribution from fermions and the nonzero Matsubara modes of bosons $p_E/T^4 \sim 1 + g^2 + g^4(1/\epsilon + 1)$, from the adjoint scalars A_0 and B_0 , $p_{M1}/T \sim m_D^3 + g_3^2 m_D^2(1/\epsilon + 1) + g_3^4 m_D(1/\epsilon + 1)$ and finally from the fundamental scalar $p_{M2}/T \sim m_3^3 + g_3^2 m_3^2(1/\epsilon + 1)$ where $m_D^2 \sim g^2 T^2$ and $m_3^2 \sim g^3 T^2$. At each step of the calculation there will be $1/\epsilon$ poles from ultraviolet and infrared divergences which will not cancel until all the contributions are summed together.

The paper is organized as follows: in section 2 we explicitly define the theory we are working with, fix various conventions and briefly review the method of dimensional reduction as applied to the present case. Sections 3 and 4 contain the essential calculations and in section 5 we discuss the result.

2. Basic setting

We consider the $SU(3)_c \times SU(2)_L \times U(1)_Y$ standard model with $n_F = 3$ families of fermions and $n_S = 1$ fundamental scalar doublets, and evaluate the pressure of this theory at temperatures slightly above the electroweak phase transition. The theory is specified by the Euclidean action (in the units $\hbar = c = 1$)

$$S = \int_0^\beta d\tau \int d^d x \mathcal{L} \tag{2.1}$$

$$\begin{aligned}
 \mathcal{L} = & \frac{1}{4}G_{\mu\nu}^a G_{\mu\nu}^a + \frac{1}{4}F_{\mu\nu} F_{\mu\nu} + \frac{1}{4}W_{\mu\nu}^a W_{\mu\nu}^a + D_\mu \Phi^\dagger D_\mu \Phi - \nu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2 \\
 & + \bar{l}_L \not{D} l_L + \bar{e}_R \not{D} e_R + \bar{q}_L \not{D} q_L + \bar{u}_R \not{D} u_R + \bar{d}_R \not{D} d_R + ig_Y \left(\bar{q}_L \tau^2 \Phi^* t_R - \bar{t}_R (\Phi^*)^\dagger \tau^2 q_L \right),
 \end{aligned} \tag{2.2}$$

where

$$\begin{aligned}
 G_{\mu\nu}^a &= \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g\epsilon^{abc} A_\mu^b A_\nu^c, & F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\
 W_{\mu\nu}^a &= \partial_\mu C_\nu^a - \partial_\nu C_\mu^a + g_s f^{abc} C_\mu^b C_\nu^c, \\
 D_\mu \Phi &= \partial_\mu \Phi - \frac{ig}{2} A_\mu^a \tau^a \Phi + \frac{ig'}{2} B_\mu \Phi, & D_\mu \Phi^\dagger &= (D_\mu \Phi)^\dagger, \\
 \not{D} l_L &= \gamma_\mu \left(\partial_\mu l_L - \frac{ig}{2} A_\mu^a \tau^a l_L + \frac{ig'}{2} B_\mu l_L \right), \\
 \not{D} e_R &= \gamma_\mu (\partial_\mu e_R + ig' B_\mu e_R), \\
 \not{D} q_L &= \gamma_\mu \left(\partial_\mu q_L - \frac{ig}{2} A_\mu^a \tau^a q_L - \frac{ig'}{6} B_\mu q_L - ig_s C_\mu^a T^a q_L \right), \\
 \not{D} u_R &= \gamma_\mu \left(\partial_\mu u_R - \frac{2ig'}{3} B_\mu u_R - ig_s C_\mu^a T^a u_R \right), \\
 \not{D} d_R &= \gamma_\mu \left(\partial_\mu d_R + \frac{ig'}{3} B_\mu d_R - ig_s C_\mu^a T^a d_R \right).
 \end{aligned} \tag{2.3}$$

Here A_μ^a , B_μ and C_μ^a are gauge bosons of weak-, hyper- and strong interactions, respectively; Φ is the fundamental scalar doublet; l_L and e_R are the left-handed lepton doublets and the right-handed lepton singlets (wrt. weak charge), and q_L , u_R and d_R are the left-handed quark doublets and the right-handed up and down -type quark singlets. The Yukawa coupling is taken into account for the top quark only. Summation over different families is assumed. Also, $d = 3 - 2\epsilon$ in dimensional regularization, $\mu, \nu = 0, \dots, d$. The gamma matrices are defined in Euclidean space so that $\{\gamma_\mu, \gamma_\nu\} = 2\delta_{\mu\nu}$, $\{\gamma_5, \gamma_\mu\} = 0$ and $\text{Tr} \gamma_5 \gamma_\mu \gamma_\nu \gamma_\rho \gamma_\sigma \propto \epsilon_{\mu\nu\rho\sigma}$. The color indices are $a = 1, \dots, d_A$ for the weak interaction and $a = 1, \dots, N_c^2 - 1$ for the strong interaction. The different group theory factors for $SU(N)$ with generators T^a are defined as:

$$T_F \delta^{ab} = \text{Tr} T^a T^b, \quad C_F \delta_{ij} = [T^a T^a]_{ij}, \tag{2.4}$$

$$C_A \delta^{ab} = f^{ace} f^{bce}, \quad d_A = \delta^{aa}, \quad d_F = \delta_{ii}. \tag{2.5}$$

For $SU(2)$ with $T^a = \tau^a/2$ they are $T_F = 1/2$, $C_F = 3/4$, $C_A = 2$, $d_A = 3$ and $d_F = 2$.

The theory in eq. (2.2) contains six couplings that run with the renormalization scale: gauge couplings g' , g and g_s , the fundamental scalar quartic self-coupling λ and its mass parameter ν^2 , and g_Y . We fix the values of these couplings at the scale $\mu = m_Z$ according to their tree-level relation with different physical parameters as in [1], where the 1-loop running of the parameters is also given. We employ a power counting rule $\lambda \sim g'^2 \sim g_s^2 \sim g_Y^2 \sim g^2$ and assume the temperature to be such that $\nu^2 \sim g^2 T^2$.

The physical observable we are studying is the pressure, defined by

$$p(T) = \lim_{V \rightarrow \infty} \frac{T}{V} \ln \int \mathcal{D}A \mathcal{D}\psi \mathcal{D}\bar{\psi} \mathcal{D}\Phi \exp(-S). \tag{2.6}$$

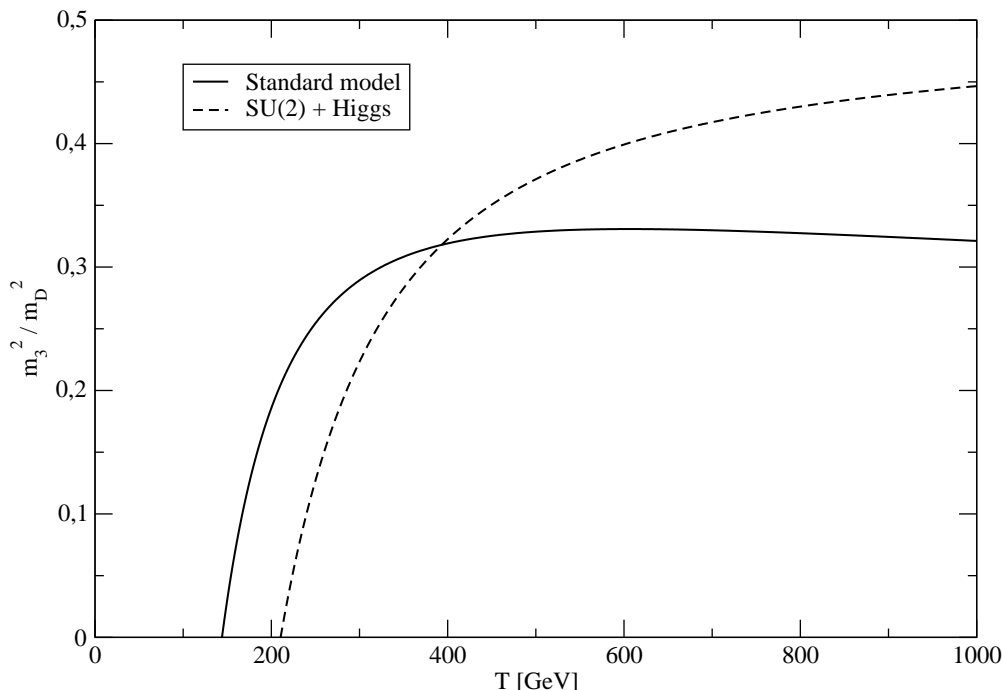


Figure 1: The ratio of the fundamental scalar thermal mass to the SU(2) adjoint scalar (Debye) mass. The expressions for the masses are given in appendix B, and the regularization scale is chosen as $\Lambda = 2\pi T$.

It is normalized such that the (real part of the) pressure of the symmetric phase vanishes at $T = 0$. Other interesting variables, such as entropy and energy densities $s(T)$ and $\epsilon(T)$, can then be evaluated using standard thermodynamic relations, $s(T) = \partial p / \partial T$, $\epsilon(T) = Ts(T) - p(T)$. The purpose is to calculate the pressure up to, and including, order $g^5(1 + \ln g)T^4$, employing the power counting rules above. For high temperatures, $g^2 T^2 \gg \nu^2$, this calculation was performed in [1]. However, close to the phase transition, which is often the most interesting region, our earlier computation is not valid, since the (thermal) fundamental scalar mass is much smaller than the Debye masses and introduces another hierarchy of scales which needs to be sorted out. The ratio of the two mass scales at different temperatures is shown in figure 1, which illustrates that there is a range of temperatures in which $m_3 \ll m_D$. The expression used for m_3^2 in figure 1 is eq. (B.4), renormalized in the $\overline{\text{MS}}$ scheme and ϵ set to 0.

It is well known that simply evaluating all 1-, 2- and 3-loop vacuum diagrams in perturbation theory fails because of infrared divergences. Instead, one has to separate the contributions of different scales into successive effective theories [11], where all the large scales are integrated out one by one. First, we integrate

$$p(T) \equiv p_E(T) + \frac{T}{V} \ln \int \mathcal{D}A_k \mathcal{D}A_0 \mathcal{D}\Phi \exp(-S_{E1}), \quad (2.7)$$

where S_{E1} contains only the static Matsubara modes of the gauge bosons and of the fundamental scalar (Higgs) field. The contributions of the nonzero Matsubara modes and

fermions to the pressure show up as the matching constant p_E (appendix A) and in the parameters of S_{E1} (appendix B). The spatial (magnetic) gauge field components remain massless, while the temporal component gets a thermal mass $m_D \sim gT$. The theory defined by S_{E1} can then be viewed as a 3d gauge theory with adjoint and fundamental scalar fields.

The effective theory thus obtained still contains contributions from several scales, $m_D^2 \sim g^2 T^2$, m_3^2 and the magnetic scale $g^2 T$. We restrict ourselves to temperatures close to the phase transition, where m_3^2 is small compared to the Debye masses and assume $m_3^2 \lesssim g^3 T^2$, motivated by eq. (4.2). We integrate out the scale gT , i.e. the fields A_0 and B_0 , and are left with

$$p(T) \equiv p_E(T) + p_{M1}(T) + \frac{T}{V} \ln \int \mathcal{D}A_k \mathcal{D}\Phi \exp(-S_{E2}). \quad (2.8)$$

This is a 3d gauge theory with a fundamental scalar field. The largest mass scale in this theory is the fundamental scalar mass $\tilde{m}_3^2 \lesssim g^3 T^2$, so this theory contributes at $\tilde{m}_3^3 \sim g^{9/2}$ or higher order, and we need to evaluate only 1- and 2-loop vacuum diagrams from this theory. The potential infrared problems can be isolated to another effective theory by integrating out the fundamental scalar,

$$p(T) \equiv p_E(T) + p_{M1}(T) + p_{M2}(T) + \frac{T}{V} \ln \int \mathcal{D}A_k \mathcal{D}\Phi \exp(-S_M). \quad (2.9)$$

The remaining effective theory contains only the (massless) spatial gauge fields. Therefore the only mass scale of the theory is provided by the 3d gauge coupling and is of the order $g^2 T$ and consequently the contribution of this theory to the pressure is of the order g^6 . The final result of our calculation can then be written as

$$p(T) = p_E(T) + p_{M1}(T) + p_{M2}(T) + p_{\text{QCD}}(T) + \mathcal{O}(g^{5.5} T^4), \quad (2.10)$$

where p_{QCD} can be taken from [2, 8–10]. One-loop quark diagrams are included in p_E , so they must be subtracted from p_{QCD} . The pressure p_E was computed in [1], and is given in appendix A, whereas p_{M1} and p_{M2} are computed in sections 3 and 4, respectively.

3. Calculation of the pressure p_{M1}

In this section we integrate over the scale gT . This means that we need to integrate out the adjoint scalars A_0 and B_0 , leaving an effective theory with only gauge bosons and the fundamental scalar field.

The 3d theory of gauge fields with fundamental and adjoint scalars is given by

$$\begin{aligned} S_{E1} = \int d^3x \left\{ \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{4} F_{ij} F_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + m_3^2 \Phi^\dagger \Phi + \lambda_3 (\Phi^\dagger \Phi)^2 \right. \\ \left. + \frac{1}{2} (D_i A_0^a)^2 + \frac{1}{2} m_D^2 A_0^a A_0^a + \frac{1}{4} \lambda_A (A_0^a A_0^a)^2 + \frac{1}{2} (\partial_i B_0)^2 + \frac{1}{2} m_D'^2 B_0 B_0 \right. \\ \left. + h_3 \Phi^\dagger \Phi A_0^a A_0^a + h_3' \Phi^\dagger \Phi B_0 B_0 - \frac{1}{2} g_3 g_3' B_0 \Phi^\dagger A_0^a \tau^a \Phi \right\}, \quad (3.1) \end{aligned}$$

where the parameters were computed in [1] to required order and are listed in appendix B.

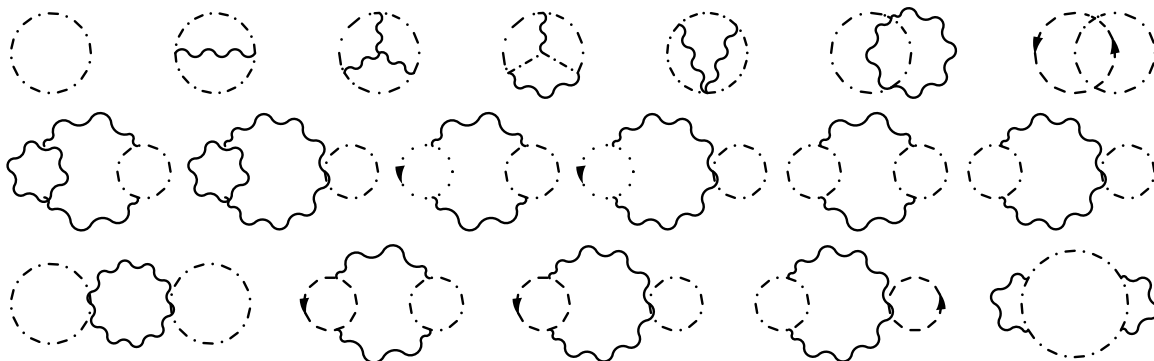


Figure 2: Diagrams contributing to p_{M1} . The dashed lines correspond to the fundamental scalar, the dot-dashed lines to the adjoint scalars, the wavy lines to the gauge fields and the dotted lines to the ghosts.

To find out the contribution of the adjoint scalars to the pressure we need to calculate all the vacuum diagrams in the theory (3.1) containing these fields. Since both A_0 and B_0 are massive, there are no infrared divergences related to this integration. The mass of the fundamental scalar m_3 is parametrically smaller ($m_3^2 \lesssim g^3 T^2$) so we treat it as a perturbation and expand the integrands in m_3^2/m_D^2 . In fact, the leading $m_3 = 0$ order is sufficient to our computations. The required diagrams are shown in figure 2 and are evaluated in [1].

The result, with $n_F = 3$ and $n_S = 1$, reads

$$\begin{aligned}
 \frac{p_{M1}(T)}{T} = & \frac{1}{4\pi} \left(\frac{1}{3} d_A m_D^3 + \frac{1}{3} m_D'^3 \right) + \frac{1}{(4\pi)^2} C_A d_A g_3^2 m_D^2 \left(-\frac{1}{4\epsilon} - \frac{3}{4} - \ln \frac{\mu_3}{2m_D} \right) \\
 & - \frac{1}{(4\pi)^3 \epsilon} \left[\frac{1}{4} C_A C_F g_3^4 m_D + d_A h_3^2 m_D + h_3'^2 m_D' + \frac{1}{4} C_F g_3^2 g_3'^2 (m_D + m_D') \right] \frac{d_F}{2} \\
 & + \frac{1}{(4\pi)^3} \left\{ g_3^4 m_D \left[C_A^2 d_A \left(-\frac{89}{24} + \frac{11}{6} \ln 2 - \frac{\pi^2}{6} \right) + C_A C_F d_F \left(-\frac{1}{2} - \frac{3}{4} \ln \frac{\mu_3}{2m_D} \right) \right] \right. \\
 & + g_3^2 g_3'^2 C_F d_F \frac{1}{4} \left[(m_D + m_D') \left(-4 - 2 \ln \frac{\mu_3}{m_D + m_D'} \right) - m_D \ln \frac{\mu_3}{2m_D} - m_D' \ln \frac{\mu_3}{2m_D'} \right] \\
 & \left. + h_3^2 m_D d_A d_F \left(-4 - 3 \ln \frac{\mu_3}{2m_D} \right) + h_3'^2 m_D' d_F \left(-4 - 3 \ln \frac{\mu_3}{2m_D'} \right) \right\}. \quad (3.2)
 \end{aligned}$$

The divergences and the scale dependence in $\mathcal{O}(g^4)$ cancel against those in p_E , and the ones in $\mathcal{O}(g^5)$ against the sunset diagram in p_{M2} , eq. (4.3).

4. Calculation of the pressure p_{M2}

The theory containing just the gauge fields and the fundamental scalar is defined by

$$S_{E2} = \int d^3x \left\{ \frac{1}{4} G_{ij}^a G_{ij}^a + \frac{1}{4} F_{ij} F_{ij} + (D_i \Phi)^\dagger (D_i \Phi) + \tilde{m}_3^2 \Phi^\dagger \Phi + \tilde{\lambda}_3 (\Phi^\dagger \Phi)^2 \right\}, \quad (4.1)$$

where the field strength tensors G_{ij}^a and F_{ij} and the covariant derivative $D_i \Phi$ are defined as before (with couplings \tilde{g}_3 and \tilde{g}_3'). The integration from S_{E1} to S_{E2} will not introduce

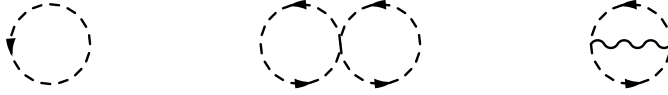


Figure 3: The diagrams required to calculate p_{M2} . The dashed lines correspond to the fundamental scalar and the wavy lines to the gauge fields.

any new corrections to the couplings to get the pressure to the required order and therefore $(\tilde{g}_3^2, \tilde{g}_3'^2, \tilde{\lambda}_3) = (g_3^2, g_3'^2, \lambda_3)$. The scalar mass does, however, get an additional contribution (calculated in [18] apart from the term $\mathcal{O}(\epsilon)$ which is needed in the present calculation) coming from the tadpole diagrams with adjoint scalars at the loops:

$$\begin{aligned} \tilde{m}_3^2 = m_3^2 - \frac{1}{4\pi} \left(d_A h_3 m_D + \frac{1}{4} g_3'^2 m_D' \right) \\ - \frac{1}{2\pi} \left[d_A h_3 m_D \left(1 + \ln \frac{\mu_3}{2m_D} \right) + \frac{1}{4} g_3'^2 m_D' \left(1 + \ln \frac{\mu_3}{2m_D'} \right) \right] \epsilon + \mathcal{O}(g^4). \end{aligned} \quad (4.2)$$

Since the pressure is to leading order given by $p_{M2}/T \sim \tilde{m}_3^3$ and since we assume the temperature to be such that $\tilde{m}_3^2 \sim g^3 T^2$, we see that corrections of order g^4 to the scalar mass would contribute to pressure at order $g^{5.5}$ and we will neglect those. Similarly, we can neglect the g^4 corrections to m_3^2 . The running of the mass m_3^2 (and hence of \tilde{m}_3^2 as well), which sets in at the order g^4 does not therefore influence the present calculation as it did the high temperature calculation. Furthermore, even though \tilde{m}_3^2 is assumed to be of the order g^3 near the phase transition, the expression for \tilde{m}_3^2 does contain individual terms of the order g^2 which, when summed up, effectively cancel to order g^3 (assuming proper temperature region). The individual terms of order g^2 must, however, be taken carefully into account when cancelling the $1/\epsilon$ poles in the pressure.

The pressure is obtained by calculating the diagrams in figure 3. Three-loop diagrams would contribute to order $g^{5.5}$ and are neglected now. The result is

$$\frac{p_{M2}}{T} = \frac{d_F}{6\pi} \tilde{m}_3^3 - \frac{\tilde{m}_3^2}{(4\pi)^2} \left[d_F(d_F + 1) \tilde{\lambda}_3 + \frac{1}{2} d_F \left(C_F \tilde{g}_3^2 + \frac{1}{4} \tilde{g}_3'^2 \right) \left(\frac{1}{\epsilon} + 3 + 4 \ln \frac{\tilde{\mu}_3}{2\tilde{m}_3} \right) \right]. \quad (4.3)$$

The $1/\epsilon$ poles cancel against those coming from p_E and p_{M1} .

5. Numerical results

In this section we will consider some numerical implications of the result, given by eq. (2.10). The convergence and the scale dependence of the perturbative expansion were already studied in [1] so now we will just concentrate on the difference between the results of that paper and the present paper, which are significant when the fundamental scalar becomes light. The Higgs mass is taken to be 130 GeV and the W mass 80 GeV.

It is instructive to first look at a simpler $SU(2) +$ fundamental Higgs model for which the perturbative expansion is better behaved. In figure 4 we have plotted the pressure of this theory in the temperature region where the fundamental scalar becomes much

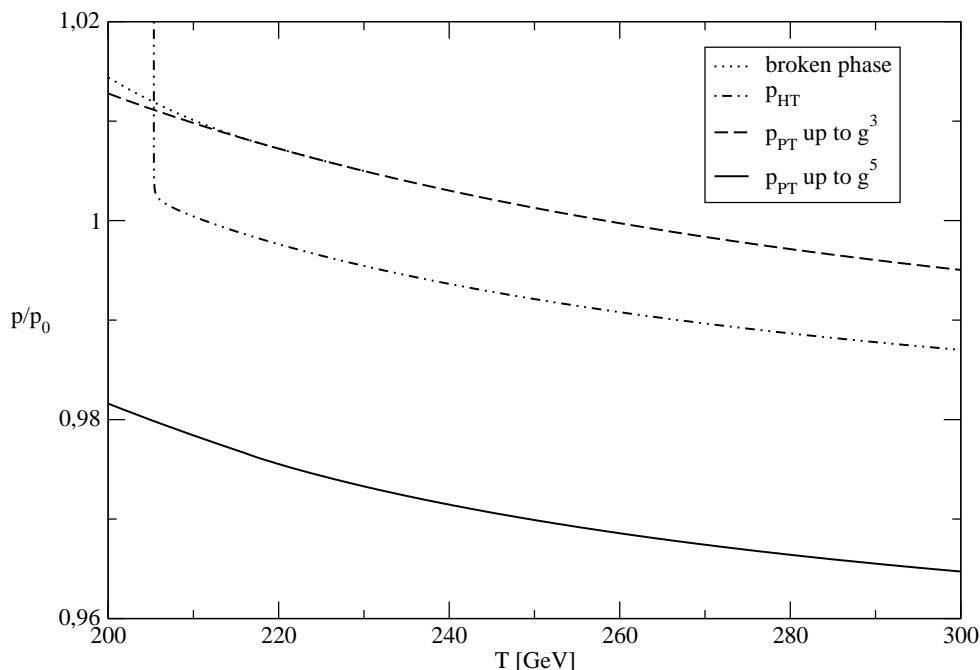


Figure 4: The pressure of the SU(2) + Higgs model, p_{PT} is the computation which takes into account that $m_3^2 \ll m_D^2$, p_{HT} is the high temperature result.

lighter than the adjoint scalars (see figure 1) using both the calculation valid at high temperatures (p_{HT}) and the calculation valid near the phase transition (p_{PT}), normalized to $p_0 = \pi^2 T^4/9$ (ideal gas pressure of SU(2) gauge bosons and a massless fundamental scalar). It can be immediately noticed that there is a temperature region where the high temperature calculation is not well behaved but becomes singular. This is due to the terms of the type $g_3^4 m_D^2/m_3$ in the expansion of p_{HT} which become singular when $m_3 \rightarrow 0$. In the present calculation which takes into account that $m_3^2 \ll m_D^2$ such terms will not appear and therefore p_{PT} is seen to be well behaved.¹ It is worth noting, however, that the temperature region where the singular behavior is manifest is very narrow. That is due to the small gauge coupling: the singular terms are of the order g^5 and their contribution to the pressure vanishes fast as one moves away from the singular point. More specifically, the singular terms behave as:

$$\frac{p_{\text{singular}}}{p_0} = \frac{135}{4096\pi^5} \frac{g^6}{\sqrt{\frac{3}{8}g^2 + \lambda}} \sqrt{\frac{T_0}{\delta T}}, \quad (5.1)$$

where T_0 is the location of the singularity and $\delta T = T - T_0$. Since the curve behaves as $\sim 1/\sqrt{\delta T}$ near the singularity and since the prefactor is so small, the contribution from the singularity becomes small very fast as δT is increased.

¹Note that as \tilde{m}_3^2 becomes negative, p_{PT} will develop an imaginary part since the symmetric phase, where the calculation is performed, is not stable anymore. The imaginary part can then be related to the rate of decay of the symmetric phase [29]. In the figures we have plotted the real part of the pressure.

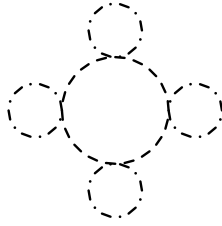


Figure 5: The type of diagrams that are resummed to the pressure p_{M2} . The dashed line corresponds to the fundamental scalar and the dot-dashed to the adjoint scalars.

If one excludes this narrow region around the singularity, the pressure p_{PT} is smaller than the pressure p_{HT} roughly by $\text{constant} \times T^4$. The relative difference between p_{HT} and p_{PT} is small, $p_{HT}/p_{PT} - 1 \sim 0.02$ but it nevertheless is of the same order of magnitude as differences in the pressure at different orders in perturbation theory and hence it is important to take this effect into account when calculating the pressure to high accuracy. This is related to the reorganization of the effective theories. Keeping the fundamental scalar mass, as defined in eq. (4.2), unexpanded when inserted to the expression for the pressure p_{M2} , eq. (4.3), effectively resums all the ring diagrams of the type shown in figure 5 to the pressure. The high temperature calculation, however, corresponds to expanding this mass in terms of $h_3 m_D/m_3^2$ in the expression for the pressure and to keeping just the lowest order terms of the expansion. At high temperatures this expansion can be made, since $h_3 m_D/m_3^2 \sim g$ but close to the phase transition $h_3 m_D/m_3^2 \sim 1$ and the expansion is not possible. Moreover, even though differences in the pressure when calculated from the high temperature theory as compared to the correct calculation near the phase transition are small, this is not necessarily the case for other interesting variables such as the critical temperature and to obtain these correctly, one needs to use the theory that takes into account that the fundamental scalar becomes light.

Although we do not study the specific features of the phase transition, which are already well established [18–27], we nevertheless plot the pressure of the broken phase as well in figure 4 to order g^3 to indicate the temperature where the phase transition takes place. In the previous sections the pressure was calculated in the symmetric phase and thus the result cannot be extended to the broken symmetry phase as such. However, we can make use of the effective potential calculations [15–17] and write $p_{BP}(\varphi) = p_{SP} - V(\varphi)$, where the effective potential is normalized so that $V(0) = 0$. The pressure in the broken phase to $\mathcal{O}(g^3)$ is then given by

$$\begin{aligned}
 p_{BP}(T, \varphi) = & \frac{1}{2} \nu^2 \varphi^2 - \frac{1}{4} \lambda \varphi^4 + \frac{\pi^2}{9} T^4 - \frac{13}{192} g^2 T^4 - \frac{1}{24} \lambda T^4 \\
 & - \frac{T^2}{24} (m_H(\varphi)^2 + 3m_{GB}(\varphi)^2 + 9m_W(\varphi)^2) \\
 & + \frac{T}{12\pi} \left[6m_W(\varphi)^3 + 3 \left(m_W(\varphi)^2 + \frac{5}{6} g^2 T^2 \right)^{3/2} \right] + \mathcal{O}(g^4), \quad (5.2)
 \end{aligned}$$

where $m_H(\varphi)^2 = 3\lambda\varphi^2 - \nu^2$, $m_{GB}(\varphi)^2 = \lambda\varphi^2 - \nu^2$ and $m_W(\varphi)^2 = g^2\varphi^2/4$ are the zero temperature masses of the particles, and $\varphi = \varphi(T)$ (the expectation value of the Higgs

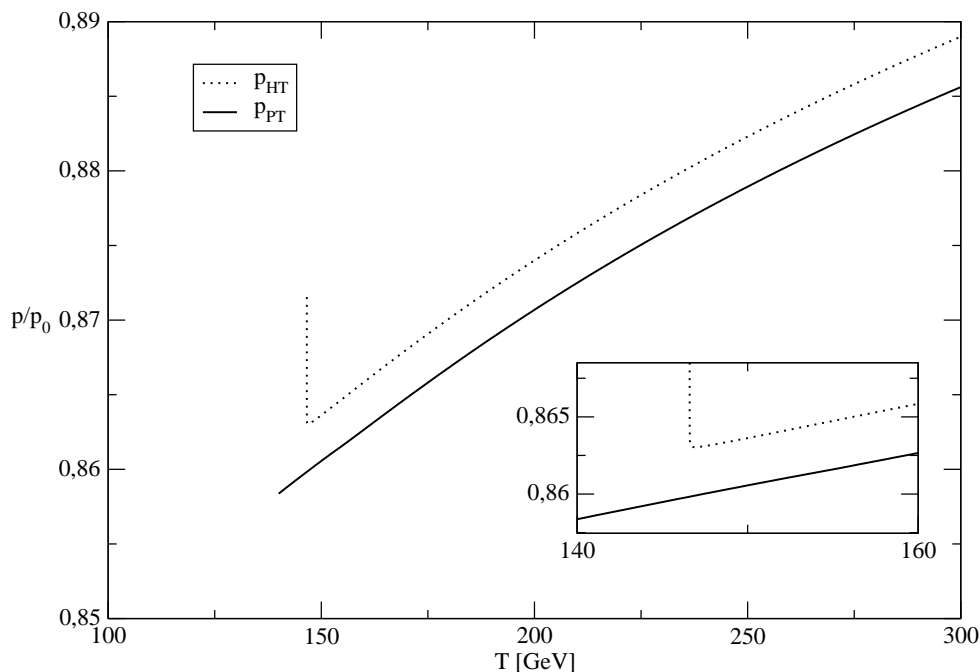


Figure 6: The pressure of the SM, p_{PT} and p_{HT} are the present and the high temperature computation, respectively.

field) is such that $\partial p_{BP}/\partial\varphi^2 = 0$. The critical temperature below which the pressure of the broken phase is bigger than the pressure of the symmetric phase, indicating that the symmetry is spontaneously broken, is, for $m_H = 130$ GeV and $m_W = 80$ GeV, about $T_c = 215$ GeV.

In figure 6 we have plotted the pressures p_{HT} and p_{PT} for the full standard model, normalized to $p_0 = 106.75\pi^2 T^4/90$. A similar structure can be seen to appear here: the high temperature calculation of the pressure develops a singularity when $m_3 \rightarrow 0$, but the temperature region where this is of importance is very narrow, while the calculation which takes into account that near the phase transition $m_3^2 \ll m_D^2$ is seen to behave well. Excluding the singularity, the pressures are again observed to differ roughly by a $\text{constant} \times T^4$, but the difference is not in this case even as large as in the $SU(2) + \text{Higgs}$ theory, especially when compared to the corrections introduced by each new order of perturbation theory. This reflects the fact that the fundamental scalar carries only a small part of the total number of degrees of freedom and hence changes in the scalar sector of the standard model do not lead to significant changes in the pressure.

6. Conclusions

In this paper we have extended the calculation of the pressure of the standard model at high temperatures carried out in [1] to the case when the temperature of the system is close to the critical temperature of the electroweak phase transition T_c . The previous calculation was not consistent in that region since it assumed that the fundamental scalar

has a mass comparable to the masses of the adjoint scalars while, in fact, near the phase transition the fundamental scalar becomes light. This inconsistency manifests itself as an unphysical singularity in the pressure if one tries to apply the result of the previous calculation to temperatures near T_c . The calculation performed in this paper is done in a consistent manner, taking into account the lightness of the fundamental scalar near the phase transition and no such unphysical singularities remain in the final result.

It is possible to go still a bit further in the perturbative expansion of the pressure, the next term would be of the order $g^{5.5}$. Its calculation would require a two-loop matching of the fundamental scalar mass (order g^4) and a three-loop calculation of the pressure p_{M2} , both of which are in principle manageable. Furthermore, one could determine the final perturbatively calculable term $g^6 \ln g$ by calculating the vacuum energy densities of the three dimensional effective theories. However, since the convergence of the perturbative expansion is rather fast (as noted in [1]), especially for the SU(2) + Higgs theory, the calculation of those terms would not lead to large numerical differences in the pressure.

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A. The pressure p_E

For completeness, we will give here the result for the pressure p_E as calculated in [1]. The general form for p_E can be written as:

$$\begin{aligned}
 p_E(T) = T^4 & \left[\alpha_{E1} + g^2 \alpha_{EA} + g'^2 \alpha_{EB} + \lambda \alpha_{E\lambda} + g_Y^2 \alpha_{EY} \right. \\
 & + \frac{1}{(4\pi)^2} \left(g^4 \alpha_{EAA} + g'^4 \alpha_{EBB} + (gg')^2 \alpha_{EAB} + \lambda^2 \alpha_{E\lambda\lambda} + \lambda g^2 \alpha_{E\lambda\lambda} + \lambda g'^2 \alpha_{EB\lambda} \right. \\
 & + g_Y^4 \alpha_{EYY} + (gg_Y)^2 \alpha_{EAY} + (g'g_Y)^2 \alpha_{EBY} + \lambda g_Y^2 \alpha_{EY\lambda} \\
 & \left. \left. + (gg_s)^2 \alpha_{EAs} + (g'g_s)^2 \alpha_{EBs} + (g_Y g_s)^2 \alpha_{EYs} \right) \right] \\
 & + \nu^2 T^2 \left[\alpha_{E\nu} + \frac{1}{(4\pi)^2} (g^2 \alpha_{EA\nu} + g'^2 \alpha_{EB\nu} + \lambda \alpha_{E\lambda\nu} + g_Y^2 \alpha_{EY\nu}) \right] \\
 & + \frac{\nu^4}{(4\pi)^2} \alpha_{E\nu\nu} + T^4 \cdot \mathcal{O}(g^6), \tag{A.1}
 \end{aligned}$$

where the coefficients α are given by:

$$\alpha_{E1} = \frac{\pi^2}{45} \left\{ 1 + d_A + d_F n_S + \frac{7}{8} \left[1 + d_F + (2 + d_F) N_c \right] n_F \right\} \tag{A.2}$$

$$\alpha_{EA} = -\frac{1}{144} \left[C_A d_A + \frac{5}{2} C_F d_F n_S + \frac{5}{4} C_F d_F (1 + N_c) n_F \right] \tag{A.3}$$

$$\alpha_{EB} = -\frac{5}{576} \left\{ \frac{1}{2} d_F n_S + \left[1 + \frac{1}{4} d_F + \left(\frac{5}{9} + \frac{1}{36} d_F \right) N_c \right] n_F \right\} \quad (\text{A.4})$$

$$\alpha_{E\lambda} = -\frac{d_F(d_F+1)}{144} n_S \quad (\text{A.5})$$

$$\alpha_{EY} = -\frac{5}{288} N_c \quad (\text{A.6})$$

$$\begin{aligned} \alpha_{EAA} = & \frac{1}{12} \left\{ C_A^2 d_A \left(\frac{1}{\epsilon} + \frac{97}{18} \ln \frac{\Lambda}{4\pi T} + \frac{29}{15} + \frac{1}{3} \gamma + \frac{55}{9} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{19}{18} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \\ & + \left[C_A C_F d_F \left(\frac{1}{2\epsilon} + \frac{169}{72} \ln \frac{\Lambda}{4\pi T} + \frac{1121}{1440} - \frac{157}{120} \ln 2 + \frac{1}{3} \gamma + \frac{73}{36} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{72} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \\ & \quad \left. \left. + C_F^2 d_F \left(\frac{35}{32} - \ln 2 \right) \right] (1 + N_c) n_F \right. \\ & + C_F T_F d_F \left(\frac{5}{36} \ln \frac{\Lambda}{4\pi T} + \frac{1}{144} - \frac{11}{3} \ln 2 + \frac{1}{12} \gamma + \frac{1}{9} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{18} \frac{\zeta'(-3)}{\zeta(-3)} \right) (1 + N_c)^2 n_F^2 \\ & + C_F T_F d_F \left(\frac{25}{72} \frac{\Lambda}{4\pi T} - \frac{83}{16} - \frac{49}{12} \ln 2 + \frac{1}{3} \gamma + \frac{1}{36} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{72} \frac{\zeta'(-3)}{\zeta(-3)} \right) (1 + N_c) n_F n_S \\ & + \left[C_A C_F d_F \left(\frac{1}{\epsilon} + \frac{317}{72} \ln \frac{\Lambda}{4\pi T} + \frac{337}{720} + \frac{2}{3} \gamma + \frac{125}{36} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{19}{72} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \\ & \quad \left. + C_F^2 d_F \left(\frac{3}{2\epsilon} + \frac{19}{2} \ln \frac{\Lambda}{4\pi T} + \frac{881}{120} + \frac{3}{4} \gamma + \frac{23}{2} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{11}{4} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \\ & \quad \left. + C_F T_F d_F \left(\frac{23}{36} \ln \frac{\Lambda}{4\pi T} - \frac{283}{360} + \frac{1}{3} \gamma + \frac{11}{18} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{11}{36} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right] n_S \left. \right\} \quad (\text{A.7}) \end{aligned}$$

$$\begin{aligned} \alpha_{EBB} = & \frac{1}{128} \left\{ \left[d_F \left(\frac{1}{\epsilon} + \frac{19}{3} \ln \frac{\Lambda}{4\pi T} + \frac{881}{180} + \frac{1}{2} \gamma + \frac{23}{3} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{11}{6} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right. \right. \\ & \left. + d_F^2 \left(\frac{23}{54} \ln \frac{\Lambda}{4\pi T} - \frac{283}{540} + \frac{2}{9} \gamma + \frac{11}{27} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{11}{54} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right] n_S \\ & + d_F \left[1 + \frac{5}{9} N_c + \frac{d_F}{4} \left(1 + \frac{N_c}{9} \right) \right] \\ & \quad \times \left[\frac{25}{27} \ln \frac{\Lambda}{4\pi T} - \frac{83}{60} - \frac{147}{135} \ln 2 + \frac{8}{9} \gamma + \frac{2}{27} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{27} \frac{\zeta'(-3)}{\zeta(-3)} \right] n_F n_S \\ & + \left[1 + \frac{17}{81} N_c + \frac{d_F}{16} \left(1 + \frac{N_c}{81} \right) \right] \left(\frac{35}{3} - \frac{32}{3} \ln 2 \right) n_F \\ & + \left[\left(1 + \frac{5}{9} N_c \right)^2 + \frac{d_F}{2} \left(1 + \frac{2}{3} N_c + \frac{5}{81} N_c^2 \right) + \frac{d_F^2}{16} \left(1 + \frac{N_c}{9} \right)^2 \right] \\ & \quad \times \left(\frac{40}{27} \ln \frac{\Lambda}{4\pi T} + \frac{2}{27} - \frac{176}{45} \ln 2 + \frac{8}{9} \gamma + \frac{32}{27} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{16}{27} \frac{\zeta'(-3)}{\zeta(-3)} \right) n_F^2 \left. \right\} \quad (\text{A.8}) \end{aligned}$$

$$\begin{aligned} \alpha_{EAB} = & \frac{1}{16} \left[C_F d_F \left(\frac{1}{\epsilon} + \frac{19}{3} \ln \frac{\Lambda}{4\pi T} + \frac{881}{180} + \frac{1}{2} \gamma + \frac{23}{3} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{11}{6} \frac{\zeta'(-3)}{\zeta(-3)} \right) n_S \right. \\ & \left. + C_F d_F \left(1 + \frac{1}{9} N_c \right) \left(\frac{35}{48} - \frac{2}{3} \ln 2 \right) n_F \right] \quad (\text{A.9}) \end{aligned}$$

$$\alpha_{E\lambda\lambda} = \frac{d_F(d_F+1)}{9} n_S \left[\ln \frac{\Lambda}{4\pi T} + \frac{31}{40} + \frac{1}{4} \gamma + \frac{3}{2} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{3}{4} \frac{\zeta'(-3)}{\zeta(-3)} + \frac{1}{4} d_F \left(\ln \frac{\Lambda}{4\pi T} + \gamma \right) \right] \quad (\text{A.10})$$

$$\alpha_{EA\lambda} = \frac{d_F(d_F+1)}{36} C_F \left(\frac{3}{\epsilon} + 15 \ln \frac{\Lambda}{4\pi T} + 11 + 3\gamma + 12 \frac{\zeta'(-1)}{\zeta(-1)} \right) n_S \quad (\text{A.11})$$

$$\alpha_{EB\lambda} = \frac{d_F(d_F+1)}{144} n_S \left(\frac{3}{\epsilon} + 15 \ln \frac{\Lambda}{4\pi T} + 11 + 3\gamma + 12 \frac{\zeta'(-1)}{\zeta(-1)} \right) \quad (\text{A.12})$$

$$\alpha_{EYY} = -\frac{1}{32}N_c \left[\ln \frac{\Lambda}{4\pi T} - \frac{239}{120} - \frac{11}{5} \ln 2 + 2 \frac{\zeta'(-1)}{\zeta(-1)} - \frac{\zeta'(-3)}{\zeta(-3)} \right. \\ \left. - N_c \left(\frac{10}{9} \ln \frac{\Lambda}{4\pi T} + \frac{53}{90} - \frac{106}{45} \ln 2 + \frac{4}{9} \gamma + \frac{4}{3} \frac{\zeta'(-1)}{\zeta(-1)} - \frac{2}{3} \frac{\zeta'(-3)}{\zeta(-3)} \right) \right] \quad (\text{A.13})$$

$$\alpha_{EAY} = \frac{1}{16}N_c \left(\frac{1}{\epsilon} + \frac{19}{4} \ln \frac{\Lambda}{4\pi T} + \frac{619}{120} - \frac{13}{4} \ln 2 + \gamma + \frac{7}{2} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{4} \frac{\zeta'(-3)}{\zeta(-3)} \right) \quad (\text{A.14})$$

$$\alpha_{EBY} = \frac{1}{48}N_c \left(\frac{1}{\epsilon} + \frac{131}{36} \ln \frac{\Lambda}{4\pi T} + \frac{6563}{1080} - \frac{41}{20} \ln 2 + \gamma + \frac{23}{18} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{49}{36} \frac{\zeta'(-3)}{\zeta(-3)} \right) \quad (\text{A.15})$$

$$\alpha_{EY\lambda} = \frac{1}{6}N_c \left(\ln \frac{\Lambda}{4\pi T} - \ln 2 + \gamma \right) \quad (\text{A.16})$$

$$\alpha_{EAs} = \frac{C_F d_F}{12} (N_c^2 - 1) n_F \left(\frac{35}{32} - \ln 2 \right) \quad (\text{A.17})$$

$$\alpha_{EBs} = \frac{1}{12} (N_c^2 - 1) n_F \left[\frac{175}{288} - \frac{5}{9} \ln 2 + \frac{d_F}{36} \left(\frac{35}{32} - \ln 2 \right) \right] \quad (\text{A.18})$$

$$\alpha_{EYs} = -\frac{15}{144} (N_c^2 - 1) \left(\ln \frac{\Lambda}{4\pi T} - \frac{62}{75} - \frac{27}{25} \ln 2 + 2 \frac{\zeta'(-1)}{\zeta(-1)} - \frac{\zeta'(-3)}{\zeta(-3)} \right) \quad (\text{A.19})$$

$$\alpha_{E\nu} = \frac{d_F}{12} n_S \quad (\text{A.20})$$

$$\alpha_{EA\nu} = -\frac{C_F d_F}{2} \left(\frac{1}{\epsilon} + 3 \ln \frac{\Lambda}{4\pi T} + \frac{5}{3} + \gamma + 2 \frac{\zeta'(-1)}{\zeta(-1)} \right) n_S \quad (\text{A.21})$$

$$\alpha_{EB\nu} = -\frac{d_F}{8} \left(\frac{1}{\epsilon} + 3 \ln \frac{\Lambda}{4\pi T} + \frac{5}{3} + \gamma + 2 \frac{\zeta'(-1)}{\zeta(-1)} \right) n_S \quad (\text{A.22})$$

$$\alpha_{E\lambda\nu} = -\frac{d_F(d_F + 1)}{3} n_S \left(\ln \frac{\Lambda}{4\pi T} + \gamma \right) \quad (\text{A.23})$$

$$\alpha_{EY\nu} = -\frac{1}{3}N_c \left(\ln \frac{\Lambda}{4\pi T} - \ln 2 + \gamma \right) \quad (\text{A.24})$$

$$\alpha_{E\nu\nu} = d_F n_S \left(\ln \frac{\nu}{4\pi T} - \frac{3}{4} + \gamma \right) \quad (\text{A.25})$$

B. Parameters of S_{E1}

For the couplings, tree level values are sufficient for our purposes:

$$\begin{aligned} g_3^2 &= g^2 T, & g_3'^2 &= g'^2 T, \\ \lambda_3 &= \lambda T, & \lambda_A &= \mathcal{O}(g^4), \\ h_3 &= \frac{1}{4} g^2 T, & h_3' &= \frac{1}{4} g'^2 T. \end{aligned} \quad (\text{B.1})$$

For the mass parameters we need terms of order $g^2(1 + \epsilon) + g^4$. These were computed in [1] and we just quote the results here. The adjoint scalars do not have divergences to this order,

$$\begin{aligned} m_D^2 &= T^2 \left[g^2 (\beta_{E1} + \beta_{E2}\epsilon + \mathcal{O}(\epsilon^2)) + \frac{g^4}{(4\pi)^2} (\beta_{E3} + \mathcal{O}(\epsilon)) + \mathcal{O}(g^6) \right. \\ &\quad \left. + \frac{g^2}{(4\pi)^2} \left(\beta_{E\lambda}\lambda + \beta_{Es}g_s^2 + \beta_{EY}g_Y^2 + \beta_{E'}g'^2 + \beta_{E\nu}\frac{-\nu^2}{T^2} \right) \right], \end{aligned} \quad (\text{B.2})$$

$$m_D^2 = T^2 \left[g'^2 (\beta'_{E1} + \beta'_{E2}\epsilon + \mathcal{O}(\epsilon^2)) + \frac{g'^4}{(4\pi)^2} (\beta'_{E3} + \mathcal{O}(\epsilon)) + \mathcal{O}(g'^6) \right. \\ \left. + \frac{g'^2}{(4\pi)^2} \left(\beta'_{E\lambda}\lambda + \beta'_{E_s}g_s^2 + \beta'_{E_Y}g_Y^2 + \beta'_{E}g^2 + \beta'_{E\nu} \frac{-\nu^2}{T^2} \right) \right]. \quad (\text{B.3})$$

The fundamental scalar, on the other hand, has $1/\epsilon$ divergences at g^4 order. The bare mass reads

$$m_3^2(\Lambda) + \delta m_3^2 = \frac{T^2}{(4\pi)^2\epsilon} \left(-\frac{81}{64}g^4 + \frac{7}{64}g'^4 + \frac{15}{32}g^2g'^2 - \frac{9}{4}\lambda g^2 - \frac{3}{4}\lambda g'^2 + 3\lambda^2 \right) \\ - \nu^2 \left[1 + \frac{g^2}{(4\pi)^2}\beta_{\nu A} + \frac{g'^2}{(4\pi)^2}\beta_{\nu B} + \frac{\lambda}{(4\pi)^2}\beta_{\nu\lambda} + \frac{g_Y^2}{(4\pi)^2}\beta_{\nu Y} \right] \\ + T^2 \left[g^2(\beta_{A1} + \beta_{A2}\epsilon) + g'^2(\beta_{B1} + \beta_{B2}\epsilon) + \lambda(\beta_{\lambda 1} + \beta_{\lambda 2}\epsilon) \right. \\ + g_Y^2(\beta_{Y1} + \beta_{Y2}\epsilon) + \frac{g^4}{(4\pi)^2}\beta_{AA} + \frac{g'^4}{(4\pi)^2}\beta_{BB} + \frac{g^2g'^2}{(4\pi)^2}\beta_{AB} + \frac{\lambda g^2}{(4\pi)^2}\beta_{A\lambda} \\ + \frac{\lambda g'^2}{(4\pi)^2}\beta_{B\lambda} + \frac{\lambda^2}{(4\pi)^2}\beta_{\lambda\lambda} + \frac{g^2g_Y^2}{(4\pi)^2}\beta_{AY} + \frac{g'^2g_Y^2}{(4\pi)^2}\beta_{BY} + \frac{g_s^2g_Y^2}{(4\pi)^2}\beta_{sY} \\ \left. + \frac{\lambda g_Y^2}{(4\pi)^2}\beta_{\lambda Y} + \frac{g_Y^4}{(4\pi)^2}\beta_{YY} \right], \quad (\text{B.4})$$

where δm_3^2 is the counter term in the $\overline{\text{MS}}$ scheme.

The constants in the above expression, with the correct group theory factors substituted into them, are as follows:

$$\beta_{E1} = \frac{2}{3} + \frac{1}{3}n_F + \frac{1}{6}n_S \quad (\text{B.5})$$

$$\beta'_{E1} = \frac{5}{9}n_F + \frac{1}{6}n_S \quad (\text{B.6})$$

$$\beta_{E2} = \frac{4}{3} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{4}{3} \ln \frac{\Lambda}{4\pi T} + \left(\frac{2}{3} - \frac{4}{3} \ln 2 + \frac{4}{3} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{4}{3} \ln \frac{\Lambda}{4\pi T} \right) \frac{n_F}{2} \\ + \left(\frac{1}{6} + \frac{1}{3} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{3} \ln \frac{\Lambda}{4\pi T} \right) n_S \quad (\text{B.7})$$

$$\beta'_{E2} = \left(\frac{5}{9} - \frac{10}{9} \ln 2 + \frac{10}{9} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{10}{9} \ln \frac{\Lambda}{4\pi T} \right) n_F + \left(\frac{1}{4} + \frac{1}{2} \frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{2} \ln \frac{\Lambda}{4\pi T} \right) n_S \quad (\text{B.8})$$

$$\beta_{E3} = \frac{20}{9} + \frac{88}{9}\gamma + \frac{88}{9} \ln \frac{\Lambda}{4\pi T} + \left(\frac{4}{9} - \frac{16}{9} \ln 2 - \frac{8}{9}\gamma - \frac{8}{9} \ln \frac{\Lambda}{4\pi T} \right) n_F^2 \\ + \left(\frac{1}{2} - \frac{32}{9} \ln 2 + \frac{28}{9}\gamma + \frac{28}{9} \ln \frac{\Lambda}{4\pi T} \right) n_F + \left(-\frac{11}{9} + \frac{13}{6}\gamma + \frac{13}{6} \ln \frac{\Lambda}{4\pi T} \right) n_S \\ + \left(\frac{1}{9} - \frac{8}{9} \ln 2 - \frac{5}{9}\gamma - \frac{5}{9} \ln \frac{\Lambda}{4\pi T} \right) n_F n_S \quad (\text{B.9})$$

$$\beta'_{E3} = \left(\frac{100}{81} - \frac{400}{81} \ln 2 - \frac{200}{81}\gamma - \frac{200}{81} \ln \frac{\Lambda}{4\pi T} \right) n_F^2 + \left(\frac{5}{72} - \frac{1}{18}\gamma - \frac{1}{18} \ln \frac{\Lambda}{4\pi T} \right) n_S \\ + \left(\frac{5}{27} - \frac{40}{27} \ln 2 - \frac{25}{27}\gamma - \frac{25}{27} \ln \frac{\Lambda}{4\pi T} \right) n_F n_S - \frac{95}{54} n_F \quad (\text{B.10})$$

$$\beta_{E\lambda} = n_S \qquad \beta'_{E\lambda} = n_S \qquad (B.11)$$

$$\beta_{E_s} = -2n_F \qquad \beta'_{E_s} = -\frac{22}{9}n_F \qquad (B.12)$$

$$\beta_{EY} = -\frac{1}{4} \qquad \beta'_{EY} = -\frac{11}{12} \qquad (B.13)$$

$$\beta_{E'} = -\frac{1}{6}n_F + \frac{1}{8}n_S \qquad \beta'_E = -\frac{1}{2}n_F + \frac{3}{8}n_S \qquad (B.14)$$

$$\beta_{E\nu} = 2n_S \qquad \beta'_{E\nu} = 2n_S \qquad (B.15)$$

$$\beta_{\nu A} = \frac{9}{2}\gamma + \frac{9}{2}\ln\frac{\Lambda}{4\pi T} \qquad \beta_{\nu B} = \frac{3}{2}\gamma + \frac{3}{2}\ln\frac{\Lambda}{4\pi T} \qquad (B.16)$$

$$\beta_{\nu\lambda} = -12\gamma - 12\ln\frac{\Lambda}{4\pi T} \qquad \beta_{\nu Y} = -12\ln 2 - 6\gamma - 6\ln\frac{\Lambda}{4\pi T} \qquad (B.17)$$

$$\beta_{A1} = \frac{3}{16} \qquad \beta_{A2} = \frac{1}{4} + \frac{3}{8}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{3}{8}\ln\frac{\Lambda}{4\pi T} \qquad (B.18)$$

$$\beta_{B1} = \frac{1}{16} \qquad \beta_{B2} = \frac{1}{12} + \frac{1}{8}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{8}\ln\frac{\Lambda}{4\pi T} \qquad (B.19)$$

$$\beta_{\lambda 1} = \frac{1}{2} \qquad \beta_{\lambda 2} = 1 + \frac{\zeta'(-1)}{\zeta(-1)} + \ln\frac{\Lambda}{4\pi T} \qquad (B.20)$$

$$\beta_{Y1} = \frac{1}{4} \qquad \beta_{Y2} = \frac{1}{2} - \frac{1}{2}\ln 2 + \frac{1}{2}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{1}{2}\ln\frac{\Lambda}{4\pi T} \qquad (B.21)$$

$$\begin{aligned} \beta_{AA} = & -\frac{17}{24} - \frac{75}{32}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{5}{32}\gamma - \frac{5}{2}\ln\frac{\Lambda}{4\pi T} + \left(\frac{1}{6} + \ln 2 - \gamma - \ln\frac{\Lambda}{4\pi T}\right)\frac{n_F}{2} \\ & + \left(-\frac{1}{12} - \frac{3}{16}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{4}\gamma - \frac{7}{16}\ln\frac{\Lambda}{4\pi T}\right)n_S \end{aligned} \qquad (B.22)$$

$$\begin{aligned} \beta_{BB} = & \frac{1}{4} + \frac{9}{22}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{3}{32}\gamma + \frac{3}{8}\ln\frac{\Lambda}{4\pi T} + \left(\frac{5}{108} + \frac{5}{18}\ln 2 - \frac{5}{18}\gamma - \frac{5}{18}\ln\frac{\Lambda}{4\pi T}\right)n_F \\ & + \left(-\frac{1}{36} - \frac{1}{16}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{1}{12}\gamma - \frac{7}{48}\ln\frac{\Lambda}{4\pi T}\right)n_S \end{aligned} \qquad (B.23)$$

$$\beta_{AB} = \frac{3}{4} + \frac{15}{16}\frac{\zeta'(-1)}{\zeta(-1)} + \frac{9}{16}\gamma + \frac{3}{2}\ln\frac{\Lambda}{4\pi T} \qquad (B.24)$$

$$\beta_{A\lambda} = -\frac{15}{4} - \frac{9}{2}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{9}{2}\ln\frac{\Lambda}{4\pi T} \qquad (B.25)$$

$$\beta_{B\lambda} = -\frac{5}{4} - \frac{3}{2}\frac{\zeta'(-1)}{\zeta(-1)} - \frac{3}{2}\ln\frac{\Lambda}{4\pi T} \qquad (B.26)$$

$$\beta_{\lambda\lambda} = 6 + 6\frac{\zeta'(-1)}{\zeta(-1)} - 6\gamma \qquad (B.27)$$

$$\beta_{AY} = -\frac{3}{16} - \frac{3}{8}\ln 2 + \frac{9}{8}\gamma + \frac{9}{8}\ln\frac{\Lambda}{4\pi T} \qquad (B.28)$$

$$\beta_{BY} = -\frac{11}{48} - \frac{55}{72}\ln 2 + \frac{17}{24}\gamma + \frac{17}{24}\ln\frac{\Lambda}{4\pi T} \qquad (B.29)$$

$$\beta_{sY} = -2 + \frac{32}{3} \ln 2 + 4\gamma + 4 \ln \frac{\Lambda}{4\pi T} \quad (\text{B.30})$$

$$\beta_{\lambda Y} = -3 \ln 2 - 6\gamma - 6 \ln \frac{\Lambda}{4\pi T} \quad (\text{B.31})$$

$$\beta_{YY} = \frac{3}{4}\gamma + \frac{3}{4} \ln \frac{\Lambda}{4\pi T}. \quad (\text{B.32})$$

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